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Boundary Layer Receptivity Theory

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110 Duncan Avenue
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submitted by
Edward J. Kerschen
Department of
Aerospace and Mechanical Engineering
The University of Arizona
Tucson, Arizona 85721

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6. AUTHOR(S)
Edward J. Kerschen

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Aerospace and Mechanical Engineering (AME)
College of Engineering and Mines
The University of Azizona
Tucson, Arizona 85721

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Edward J. Kerschen

Principal Investigator

Final Report

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SUMMARY

Receptivity processes by which free-stream disturbances generate Tollmien-Schlichting waves in boundary layers have been analyzed using asymptotic methods for high Reynolds numbers. Vortical and acoustic free-stream disturbances have been considered. Receptivity occurs in the vicinity of the leading edge, and in localized regions further downstream where some feature (e.g., a wall hump) produces a short-scale disturbance to the mean flow. Nonlinear effects related to the free-stream pressure field have been found to play an important role in localized receptivity to vortical disturbances. For leading-edge receptivity, the influences of the body nose radius and aerodynamic loading in the leading-edge region have been analyzed. In the absence of aerodynamic loading, an increase in the body nose radius decreases the leading-edge receptivity coefficient. However, strong aerodynamic loading leads to a dramatic increase in the leading-edge receptivity coefficient, negating the decrease due to a larger nose radius. The propagation of an instability wave past a junction between a rigid wall and a surface with non-zero compliance or admittance has also been analyzed. The junction can cause energy to be scattered from the instability wave to higher eigenmodes, effectively attenuating the instability wave.

1. INTRODUCTION

Receptivity is a term used to describe the processes by which instability waves are generated in wall-bounded and free shear layers. The understanding of these processes is important in a number of technological applications. Instability waves have obvious relevance in the process of transition to turbulence, and are also closely linked to the large-scale structures that control many features of turbulent flows. The latter connection is especially strong for flows that are dominated by inflectional instabilities. Approaches to minimize the generation of instability waves are of interest in situations where it is desirable to delay transition. On the other hand, in order to actively control certain transitional or turbulent flows, it may be advantageous to maximize the generation of instability waves. To most efficiently achieve either of these objectives, an understanding of receptivity processes is required.

Natural receptivity[1] refers to the generation of instability waves by naturally occurring external disturbances, such as free-stream sound waves or vortical structures. Forced receptivity[1] concerns the generation of instability waves by artificially produced, generally localized disturbances. An essential feature for the generation of instability waves is the presence of a short, local, streamwise length scale that is commensurate with the wavelength of the instability wave. In the case of natural receptivity, a short local length scale in the mean flow allows energy to be transferred from the long wavelength of the free-stream disturbance to the much shorter wavelength of the instability wave. Natural receptivity mechanisms occur in the leading-edge region, and in any localized region further downstream where some feature causes a short-scale adjustment in the mean flow. For forced receptivity, the short local length scale is generally present in the artificially introduced disturbance. Finally, short local length scales in the mean flow or boundary conditions can also cause a scattering of an instability wave that is already present in a boundary layer. This scattering process can be an effective mechanism for attenuating instability waves. In the present research project, theoretical models have been developed for natural receptivity, forced receptivity, and for instability wave scattering by short-scale changes in wall boundary conditions. These analyses are discussed in the following section.

2. RESEARCH ACCOMPLISHMENTS

The objective of this research project was to develop theoretical analyses for receptivity processes in boundary layers. Asymptotic methods have been utilized to develop singular perturbation solutions for high Reynolds numbers. Theoretical models have been developed for several types of boundary-layer receptivity processes. These are: (a) localized receptivity to vortical free-stream disturbances, (b) influences of body nose radius and aerodynamic loading on leading-edge receptivity, (c) instability wave attenuation by scattering due to step changes in wall admittance or compliance, and (d) forced receptivity to artificially generated free-stream disturbances. The results of the research on each of these topics is summarized below. Additional information can be found in the references.

2a. Localized Receptivity to Vortical Free-Stream Disturbances

Localized receptivity mechanisms occur some distance downstream of the leading edge, in regions where some feature causes a short-scale adjustment to the mean boundary-layer flow. A prototypical problem in this class of natural receptivity mechanisms is receptivity due to a localized wall hump. Goldstein[2] and Ruban[3] independently analyzed receptivity due to acoustic waves interacting with a short-scale variation in wall geometry. The variation in wall geometry was assumed to have a length scale of $O(\epsilon^3 L)$, where $\epsilon = (U_\infty L/\nu)^{-1/8}$ and U_∞, L and ν are the free-stream speed, the distance from the leading edge, and the kinematic viscosity of the fluid, respectively. The short-scale adjustment in the mean flow is described by the triple-deck structure. The unsteady flow also has the triple-deck structure, and the solutions show that the interaction of the free-stream sound wave with the short-scale mean flow adjustment leads to the generation of a Tollmien-Schlichting wave. Choudhari and Kerschen[4] extended the analysis of localized receptivity to acoustic free-stream disturbances to the three-dimensional case.

In the present research, localized receptivity analyses have been developed for the case of vortical free-stream disturbances interacting with a wall hump. The Mach number of the flow is assumed to be small, and high-Reynolds-number singular perturbation techniques are utilized. In all cases the hump height is assumed small, say $\delta << 1$. The case of small roughness heights is applicable to many practical situations, and has the advantage that analytical solutions to the triple-deck equations can then be found. A theory that is also linear in the amplitude A of the vortical free-stream disturbance was developed first. This theory showed that localized receptivity to vortical disturbances is significantly weaker than the localized receptivity to acoustic disturbances. However, the results of the theory also suggested that, even at small amplitudes, nonlinear effects could be important for localized receptivity to vortical disturbances. Therefore, the localized receptivity theory was extended to account for nonlinear effects that arise at larger free-stream disturbance amplitudes. The essential features of these theories are described below.

First consider the theory that is linear in the amplitude A of the vortical free-stream disturbance. Under the linear approximation, vortical free-stream disturbances are convected at the speed U_∞ of the free-stream. This linear disturbance field can be represented by a superposition of convected gusts,

$$\mathbf{v} = \mathbf{A} \exp\,(ik_x x + ik_y y - ik_x U_\infty t)\ ,$$

where $\mathbf{A}\cdot\mathbf{k} = 0$, since the disturbance field is solenoidal. Thus, the velocity fluctuations are perpendicular to the wavevector. There are no pressure perturbations associated with the linear convected gust. This result can be seen from the linearized momentum equation upon noting that the vortical disturbance convects at speed U_∞. The absence of free-stream pressure perturbations for a linearized vortical free-stream disturbance has important consequences for the localized receptivity, as will be seen below.

The solution takes the form of a double perturbation series in the small parameters δ and A,

$$u(x,y,t) = U(x) + \delta\, u_1(x,y) + A\, u_2(x,y,t) + \delta A\, u_3(x,y,t) + \dots\ ,$$

where U is the undisturbed boundary layer flow at the location of the wall hump, u_1

is the mean flow perturbation due to the wall hump and u_2 is the base unsteady flow that exists in the absence of the wall hump. The $O(\delta A)$ term u_3 arises from the interaction of the short-scale mean flow disturbance u_1 with the unsteady flow u_2; it is this term that contains the instability wave generated by the localized receptivity process.

The source terms that produce u_3 contain products of the mean-flow disturbance u_1 and the base unsteady flow u_2. Therefore it is clear that the characteristics of u_2 have an important influence on the receptivity level. The base unsteady flow u_2 for the vortical free-stream disturbance turns out to be significantly different than that for an acoustic free-stream disturbance. In order to explain this difference, it is helpful to first consider the case of an acoustic free-stream disturbance.

For an acoustic free-stream disturbance, the unsteady pressure field drives an inviscid velocity perturbation throughout the main part of the boundary layer. This inviscid velocity perturbation is brought to rest at the wall by a viscous Stokes layer whose thickness is of the same order as the lower deck of the triple-deck, i.e., $O(\epsilon^5 L)$. The receptivity arises from the interaction of the mean flow disturbance u_1 with the Stokes wave component of u_2 in the lower deck.

In contrast, a linear vortical free-stream disturbance does not contain a pressure field. Thus, the free-stream disturbance can penetrate the boundary layer only through viscous diffusion. This process is very weak at high Reynolds numbers, and therefore the unsteady disturbance u_2 within the boundary layer is exponentially small in the Reynolds number parameter ϵ. Thus, the receptivity process does not arise from the direct interaction of u_1 and u_2 in the lower deck, but rather from a more indirect route involving an interaction in the upper deck. Essentially, the wall hump produces short-scale mean flow gradients in the upper deck outside the boundary layer. These mean flow gradients just outside the boundary layer distort the vortical disturbance, generating pressure fluctuations in the process. These pressure fluctuations penetrate the boundary layer and generate Tollmien-Schlichting waves. However, since the process is less direct than for the case of an acoustic free-stream disturbance, the receptivity level for the vortical free-stream disturbance turns out to be weaker than that for an acoustic wave by one factor of ϵ.

The efficiency functions for localized receptivity to acoustic and linear vortical (convected gust) free-stream disturbances are plotted as a function of the triple-deck frequency parameter $S = \epsilon^2 \omega L/U_\infty$ in Fig. 1. Except at low frequencies, the efficiency function for the convected gust is smaller than that for the acoustic wave. Combining the smaller numerical value of the efficiency function with the additional factor of ϵ discussed in the preceding paragraph, the localized receptivity to a convected gust turns out to be significantly weaker than the receptivity to an acoustic wave, by a factor of as much as 15 for a wall hump at $Re = 10^6$. Thus, the absence of a free-stream pressure field has a profound influence on localized receptivity to linear vortical free-stream disturbances.

At second order in the disturbance amplitude A, vortical free-stream disturbances generate pressure fluctuations that can produce Tollmien-Schlichting waves through the more efficient "acoustic" mechanism for localized receptivity. This implies that, even at fairly small disturbance amplitudes, nonlinear effects may be important in localized receptivity to vortical free-stream disturbances. Thus, the localized receptivity theory was extended to incorporate the influence of nonlinearities in the free-stream disturbance field. An interesting feature of the analysis is that, since the vortical disturbances convect at a speed close to the free-stream speed, the nonlinearity is concentrated outside the boundary layer. The unsteady motion within the boundary layer remains linear.

The relative importance of the contribution to localized receptivity due to the nonlinear pressure field depends on specific features of the vortical free-stream disturbance. Several examples are considered in Ref. 5. In Fig. 2, results are presented for the case of receptivity to a Karman vortex street passing above a localized wall hump. Due to nonlinear effects, the vortex street convects at a speed U_c that is different than the free-stream speed U_∞. A normalized receptivity level is plotted as a function of $c = U_c/U_\infty$ in Fig. 2, the receptivity level for $c = 1$ being used to normalize the ordinate. It can be seen that the receptivity level drops sharply with a slight decrease in the convection speed, reaching a minimum which is less than 10% of the linear result for $c = U_c/U_\infty = 0.94$. This minimum is due to cancellation between the linear "convected gust" receptivity due to the slip velocity of the disturbance and the nonlinear "acoustic" receptivity due to the pressure field. With further decreases in the convection speed, the receptivity rises rapidly due to the increased value of the nonlinear pressure field. A rapid rise also occurs as $c = U_c/U_\infty$ increases above one, again due to the second-order pressure field. This dramatic sensitivity of the receptivity level to the convection speed of the vortical free-stream disturbance illustrates that the nonlinear free-stream pressure field can be very important for localized receptivity to vortical free-stream disturbances. Further details on this work may be found in Ref. 5.

2b. Influences of Body Nose Radius and Aerodynamic Loading on Leading-Edge Receptivity

In leading-edge receptivity, the mean flow gradients associated with the rapid boundary layer development near the body nose cause a transfer of energy from the long-wavelength free-stream disturbance to the short-wavelength instability wave. The general asymptotic structure of the leading-edge receptivity problem was elucidated by Goldstein[6], who considered the Blasius boundary layer on a flat plate. His theory can be considered as the limiting case of a body whose nose radius is small compared to the hydrodynamic length scale U_∞/ω for the leading-edge receptivity process. Goldstein *et al*[7] calculated the leading-edge receptivity coefficient for an acoustic wave propagating parallel to the surface of the flat plate. Heinrich and Kerschen[8] presented leading-edge receptivity coefficients for a variety of free-stream disturbances interacting with the boundary layer on a flat plate.

The research described in the preceding paragraph made important contributions to the understanding of leading-edge receptivity, but had a number of limitations. In many applications of interest, the body nose radius is not small compared to the hydrodynamic length scale U_∞/ω. Furthermore, aerodynamic loading is present on most aeronautical bodies. The aerodynamic loading is known to have an important influence of boundary layer stability, and therefore would be expected to significantly influence the receptivity process as well. Unfortunately, aerodynamic loading cannot be incorporated in a receptivity theory that utilizes the flat-plate geometry, since any flow around the sharp leading edge causes boundary layer separation.

In order to overcome these limitations, a theory has been developed for leading-edge receptivity on a body with a finite nose radius r_n. The development of this theory is a substantial effort that was carried out jointly by the author and Dr. Paul

W. Hammerton, a post-doctoral student who was supported by funding from the NASA Langley Research Center. The flow is assumed to be a low Mach number, two-dimensional flow, and high-Reynolds-number asymptotic methods are employed in the analysis. The small parameter ϵ_1 in the asymptotic theory is an inverse of the Reynolds number based on the hydrodynamic length scale U_∞/ω, $\epsilon_1 = (\omega\nu/U_\infty^2)^{-1/6}$. Aerodynamic bodies generally have a parabolic shaped nose region, and hence the leading-edge receptivity problem reduces to the case of flow past a semi-infinite parabola. The Strouhal number based on the body nose radius, $S = \omega r_n/U_\infty$, is assumed to be an $O(1)$ parameter.

Two distinct streamwise regions are involved in the leading-edge receptivity analysis. The appropriate streamwise length scale near the leading edge is the hydrodynamic length scale U_∞/ω. The unsteady flow in the leading-edge region can be separated into an inviscid flow outside the mean boundary layer, and a viscous flow inside the boundary layer. The unsteady disturbances in the boundary layer are governed by the Linearized Unsteady Boundary Layer Equation (LUBLE). An important feature of the LUBLE is that the streamwise gradients of the mean flow enter at leading order. A second streamwise region is present further downstream where $\omega x/U_\infty = O(\epsilon_1^{-2})$. The unsteady disturbances in the latter region satisfy the large-Reynolds-number, small-wavenumber approximation to the Orr-Sommerfeld Equation (OSE), or equivalently, the triple-deck structure.

A key aspect of the leading-edge receptivity analysis is the asymptotic matching between the LUBLE and OSE regions. For large values of $\omega x/U_\infty$, the LUBLE admits an infinite set of asymptotic eigenfunctions that decay exponentially with downstream distance. The asymptotic matching of the LUBLE and OSE regions shows that the first of this infinite set of eigenfunctions matches to the eigenfunction of the OSE corresponding to the Tollmien-Schlichting wave. Thus, the first asymptotic eigenfunction of the LUBLE is the precursor of the TS wave. The leading-edge receptivity problem then reduces to the determination of the coefficient, say C_1, of this first asymptotic eigenfunction.

The Tollmien-Schlichting wave near the lower-branch neutral curve is then given by

$$u_{TS} = u_\infty \, C_1 \, h(\omega) \, f(y) \, e^{i(\kappa x - \omega t)} .$$

Here u_∞ is the amplitude of the free-stream disturbance, $h(\omega)$ is a function of frequency which effectively represents damping of the TS wave over the distance from the leading edge to the lower-branch neutral point, and $f(y)$ and κ are the mode shape and wavenumber of the TS wave. The leading-edge receptivity coefficient C_1 is a function of the free-stream disturbance characteristics, the body nose radius parameter $S = \omega r_n/U_\infty$, and an aerodynamic loading parameter μ, defined below.

Solutions of the LUBLE cannot, in general, be found analytically. Thus, to determine the leading-edge receptivity coefficient C_1, a numerical solution of the LUBLE is required. Essentially, the $\omega x/U_\infty \gg 1$ behavior of the numerical solution is compared to the analytical expression for the first asymptotic eigensolution. However, for real values of x, the first asymptotic eigensolution makes only an exponentially small contribution to the $\omega x/U_\infty \gg 1$ behavior of the solution. Therefore, in order to extract C_1 from the behavior of the numerical solution, the equation is analytically continued into the complex x plane and solved along a path where the first asymptotic eigensolution dominates at large values of $|x|$.

The influence of body nose radius on the leading-edge receptivity coefficient for a symmetric mean flow past the parabolic leading edge is illustrated in Fig. 3. The dependence of C_1 on the nose radius can be expressed in terms of the Strouhal number $S = \omega r_n / U_\infty$. For small values of S, the result reduces to that found previously for the Blasius boundary layer on a flat plate. It can be seen from Fig. 3 that the leading-edge receptivity coefficient decreases monotonically as S increases. The reduction in leading-edge receptivity with increasing nose radius is quite dramatic. At $S = 0.3$, the receptivity coefficient has decreased by a factor of 6 relative to that for the Blasius boundary layer ($S = 0$).

The physical explanation for the decrease in the leading-edge receptivity coefficient with increasing body nose radius lies in the favorable pressure gradient on the surface of the parabola. The pressure gradient at the body nose is highly favorable, corresponding to the value for stagnation point flow. The pressure gradient gradually relaxes to zero as one moves downstream along the body surface, the length scale for this decay being the body nose radius. Favorable pressure gradients are known to cause boundary layer stabilization, decreasing instability wave growth. The favorable pressure gradient exerts a similar damping effect on the precursor to the Tollmien-Schlichting wave, leading to a decrease in the receptivity coefficient C_1. As S increases the strongly favorable pressure gradient near the nose extends over a larger number of disturbance wavelengths, and this additional stabilizing influence results in a decrease in the leading-edge receptivity coefficient. Additional details on the leading-edge receptivity analysis for a symmetric mean flow can be found in Refs. 9 and 10.

The leading edge receptivity theory has also been extended to incorporate the influence of mean aerodynamic loading in the leading edge region. The presence of aerodynamic loading is generally accompanied by an asymmetric component of mean flow around the leading edge from the lower to the upper surface. The strength of the mean flow component around the airfoil leading edge can be expressed in terms of an effective leading-edge incidence angle[11] α_{eff}, which depends on both the camber distribution and the angle of attack of the airfoil. The boundary layer behavior in the leading edge region depends on the parameter $\mu = \alpha_{eff}\sqrt{b/r_n}$, where b is the airfoil semi-chord. The dependence on the single parameter μ can be understood by noting that the airfoil nose radius r_n is the only length scale in the leading edge region. The parameter μ is a measure of the asymmetric displacement of the mean flow stagnation point, normalized by r_n. For values of μ exceeding approximately 1.1, separation of the laminar boundary layer on the upper surface takes place.

The influence on the receptivity coefficient of aerodynamic loading in the leading edge region is illustrated in Fig. 4. Results are shown for three different values of the nondimensional nose radius, S. As μ is increased from zero, the receptivity coefficient decreases at first and then rises sharply. This behavior can also be related to pressure gradient effects. For small values of μ, the favorable pressure gradient in the region of receptivity near the body nose becomes stronger, while for larger values of μ the destabilizing adverse pressure gradient on the airfoil upper surface moves into the region where the receptivity occurs. The present theory is not valid for boundary layers that undergo separation, and the numerical computations become increasingly more difficult as μ approaches the value corresponding to separation (approximately 1.1). However, the results clearly show that the leading-edge receptivity becomes quite strong as the aerodynamic loading increases toward the value at which the boundary layer separates. Further details on the influence of aerodynamic loading on leading-edge receptivity can be found in Ref. 12.

This work has concentrated on the receptivity of the boundary layer rather than on its stability. The transition process is controlled by the combined influences of receptivity and stability. In this context it is worth noting that the influences of body nose radius and aerodynamic loading on leading-edge receptivity and on stability are complementary. Specifically, an increase in body nose radius in the absence of aerodynamic loading decreases the receptivity coefficient, and also decreases instability wave growth. On the other hand, strong aerodynamic loading increases the receptivity coefficient and also increases instability wave growth rates. Thus, the influence of stability works in concert with the leading-edge receptivity to magnify the effects of body nose radius and aerodynamic loading on the boundary layer transition process.

2c. Instability Wave Attenuation by Scattering due to Step Changes in Wall Admittance or Compliance

A common feature of the receptivity mechanisms discussed above is the presence of a short-scale streamwise inhomogeneity. In the receptivity mechanisms, the short-scale inhomogeneity causes a transfer of energy from the free-stream disturbance to the instability wave. Short-scale streamwise inhomogeneities also can significantly affect an instability wave that is already present in the boundary layer. In particular, a sudden change in wall boundary condition can scatter energy out of the Tollmien-Schlichting wave mode and into the higher eigenmodes of the boundary layer. These higher eigenmodes are strongly damped and decay rapidly with downstream distance. This suggests that mode scattering has promise as an approach for attenuating instability waves.

Mode scattering could be produced by any short-scale change in surface properties, such as a wall hump or a suction slot, for example. However, wall humps have the undesirable property that the boundary layer flow downstream of the hump is often inflectional, leading to an increase in instability wave growth rates. Surface suction is attractive in that it generally decreases instability wave growth rates. However, the implementation of suction introduces significant additional complexities in practical applications. In comparison, the implementation of local surface treatments appears more straightforward. In the present research, mode scattering due to strips of a compliant or admittance surface embedded in a rigid wall has been investigated.

A substantial body of literature has shown that properly designed compliant surfaces are effective in damping the growth of instability waves. The compliant surface can be modeled as a continuous spring-mass-damper system. The surface sheet represents a mass, and is often modeled as a flexible plate, sometimes under tension, supported by a spring-damper system. An alternative to a compliant surface that has received less attention is an admittance surface. An admittance surface usually consists of a rigid but porous surface sheet, backed by an acoustic cavity. The admittance surface can also be related to a spring-mass-damper system. The mass in the system relates to the inertia of the fluid passing through the porous sheet. The spring constant relates to compression/expansion of the fluid in the backing cavity, and damping occurs due to viscous effects in the fluid passing through the porous sheet. Admittance surfaces are commonly used as acoustic treatment in aircraft engine inlets and outlets.

Previous research on compliant and admittance surfaces has focused on the stability of the boundary layer, and has assumed that the compliant or admittance surface is of infinite extent. The present research investigates the effects of a step change in surface properties. The theory assumes that the flow is two-dimensional and the Mach number is small. The analysis utilizes the triple-deck theory, a large-Reynolds-number asymptotic structure based on the small parameter $\epsilon = (U_{\infty} L/\nu)^{-1/8}$, where L is the distance from the leading edge to the step change in wall property.

On the admittance surface, the no-penetration boundary condition $v' = 0$ is replaced the admittance condition, $v' = -\beta p'$, where β is the surface admittance. The boundary condition for the compliant surface is specified in terms of the differential equation describing the motion of the surface sheet. The following discussion focuses mainly on the case of admittance surfaces; in most respects the results for compliant surfaces are similar. The differences that arise between admittance and compliant surfaces are noted in the following discussion.

Solutions have been developed using two different methods. A perturbation expansion for small values of the surface admittance was developed first. In principle, this approach can treat arbitrary spatial distributions of the surface admittance. However, concentration was placed on the cases of a step change from a rigid, impermeable wall to a treated surface, a step change from a treated surface to a rigid wall, and a finite-length strip of treated surface. The perturbation theory showed that the instability wave amplitude is modified by mode scattering. However, for the leading term of the perturbation expansion which is linear in β, a decrease in instability wave amplitude due to mode scattering at the front edge of a treated strip is cancelled by an increase in instability wave amplitude due to mode scattering at the rear edge of the strip. Thus, the perturbation theory must be carried out to second order in β in order to obtain a net change in instability wave amplitude due to mode scattering by a finite-length strip. The second-order perturbation theory shows that mode scattering can lead to a net reduction in the amplitude of the instability wave. However, since the reduction is proportional to β^2, mode scattering is not important for small values of β. Additional results on the perturbation theory can be found in Ref. 13.

The second method used to develop solutions was the Wiener-Hopf technique. This technique can treat $O(1)$ values of the surface admittance, but is applicable only to two-part boundary-value problems. Thus, cases that can be analyzed by the Wiener-Hopf technique include a step change from an impermeable wall to a treated surface, and vice-versa. An approximate solution for sufficiently wide strips can be obtained by combining the Wiener-Hopf solutions for the front and rear edges of the strip. The Wiener-Hopf technique is based on analytic continuation in the complex plane, the relevant complex variable being the wavenumber of the Fourier transform with respect to the streamwise coordinate. The solution is found in terms of contour integrals in the complex plane. Thus, it is necessary to determine the full analytic structure of the instability wave dispersion relationships in the complex wavenumber plane, for both the treated and untreated surfaces.

Results of the Wiener-Hopf analysis are presented in Figs. 5 and 6. Mode scattering at a junction between an upstream impermeable surface and a downstream admittance surface with $|\beta| = 1.3$ is illustrated in Fig. 5. The quantity plotted is the amplitude of the Tollmien-Schlichting wave. For the upstream impermeable surface, the wave is unstable and its amplitude increases with x. The wave on the admittance surface is damped and therefore decays with x. Due to mode scattering, the Tollmien-Schlichting wave exhibits a discontinuous change in amplitude at the junction. It should be noted that the complete velocity and pressure field is continuous across the junction. However, since the upstream and downstream wall properties are different, the Tollmien-Schlichting mode shapes in the upstream and downstream regions are also different. This difference in mode shapes leads to the mode scattering that can result in a decrease of the Tollmien-Schlichting wave amplitude.

Results for the change in Tollmien-Schlichting wave amplitude due to mode scattering at junctions between an impermeable surface and an admittance surface are presented in Fig. 6. The quantity plotted is $(|A_+| - |A_-|)/|A_-|$, the change in the amplitude of the TS wave normalized by the amplitude just upstream of the junction, as a function of $|\beta|$, the magnitude of the wall admittance. Results are presented both for a change from a rigid surface to an admittance surface and for a change from an admittance surface to a rigid surface.

First consider the case of a change from a rigid surface to an admittance surface. For the particular value of $Arg(\beta)$ utilized in Fig. 6, small values of $|\beta|$ lead to an increase in the TS wave amplitude. However, as $|\beta|$ increases this trend reverses, and for $|\beta| > 1.1$ mode scattering at the junction significantly decreases the TS wave amplitude. Next consider the case of a change from an admittance surface to a rigid surface. For this case mode scattering causes a monotonic decrease in the TS wave amplitude, out to $|\beta| = 1.25$. Beyond this value of $|\beta|$ there is a gradual reduction in the effectiveness of mode scattering in attenuating the TS wave. Combining the results for the two cases, it can be seen that for values of $|\beta| > 1$ the mode scattering at both edges of a finite length strip attenuates the instability waves. These results suggest that an array of admittance surface strips placed in a rigid wall could be very effective in attenuating TS waves, due to the mode scattering that would occur at each of the junctions in the array.

The perturbation and Wiener-Hopf analyses have also been developed for the case of compliant surfaces. The general features of the results are similar to those for the admittance surfaces described above. However, one new feature arises in the compliant surface problem. This is the influence of the edge conditions that specify how the face sheet of the compliant surface is attached to the rigid surface. The connection of the face sheet to the rigid wall may correspond to a clamped or pinned joint, or the edge of the face sheet may be free to move vertically at the junction location $x = 0$. The edge conditions have been found to have an important influence on the mode scattering process for face sheets that have significant bending resistance or are under significant tension. Additional details on the Wiener-Hopf solutions for mode scattering at junctions between rigid, impermeable walls and compliant or admittance surfaces can be found in Ref. 14.

2d. Forced Receptivity to Artificially Generated Free-Stream Disturbances

As one approach to studying receptivity processes, experimenters have examined the generation of boundary layer instability waves by artificially introduced free-stream disturbances. Nishioka and Morkovin[15] studied the receptivity produced by a concentrated, time-harmonic volume source located just outside the boundary layer.

Physically, the volume source was produced by an unsteady flux through the open end of a small-diameter tube. Kendall[16] generated free-stream disturbances by rotating a small-diameter rod on a circular path outside the boundary layer. The rod passed near the boundary layer during the bottom-dead-center portion of the circular path, sweeping by in the downstream direction.

These two experiments have been used to support the concept of a "distributed" receptivity mechanism, in which the receptivity is assumed to be caused by gradients of a free-stream disturbance that extend over a distance long compared to the instability wavelength. The "distributed" receptivity mechanism is assumed to be present for a body with a smooth surface and a nearly-parallel boundary layer. It has been suggested that this "distributed" receptivity mechanism is important for applications such as airfoils, where the curvature of the body surface leads to gradients in the free-stream sound field.

The above concept of a "distributed" receptivity mechanism has been investigated analytically in work carried out jointly with Dr. T. B. Gatski of NASA Langley Research Center. Triple-deck theory was utilized to develop models of: (a) the two-dimensional analogue of Nishioka and Morkovin's pulsating-source experiment, and (b) Kendall's rotating-rod experiment. These theoretical models show that streamwise gradients of free-stream disturbances can cause receptivity. However, the receptivity level is significant only for free-stream disturbances whose spatial gradients occur on the short length scale of the instability wave.

Specifically, the length scale of the free-stream disturbance for the Nishioka and Morkovin experiment is h, the distance of the concentrated source from the body surface. The present theory shows that the receptivity level is proportional to $\exp(-\kappa_0 h)$, where κ_0 is the complex wavenumber of the Tollmien-Schlichting wave generated by the harmonically pulsating source of frequency ω_0. For the Kendall experiment, a wavepacket is generated each time the rotating rod sweeps by the boundary layer. The present theory shows that the wavepacket amplitude is proportional to $\exp(-\kappa_m h_0)$, where h_0 is the height of the rod above the surface of the plate at its point of closest approach, and κ_m is the wavenumber of the maximally growing Tollmien-Schlichting wave.

As one illustration that the theoretical models capture the essential features of the receptivity processes, predictions of the theory are compared to Kendall's experimental results in Fig. 7. Figure 7*a*, reproduced from Ref. 16, is a measurement of the instability wave amplitude as a function of the speed U_c at which the rod sweeps by the boundary layer, normalized by the free-stream speed. The instability wave amplitude predicted by the present theory is shown in Fig. 7*b*. The ordinates of the two curves cannot be compared directly, since Fig. 7*a* is a narrowband measurement while Fig. 7*b* is based on the maximum velocity fluctuation within the wavepacket. However, the difference in ordinates does not influence the dependence on the rotor speed U_c. Kendall's experimental results (Fig. 7*a*) show that the maximum receptivity occurs for a rod speed of approximately $0.4\,U_\infty$. The prediction of the present theory (Fig. 7*b*) for the influence of the rotor speed U_c is in remarkably good agreement with Kendall's measurements. This comparison illustrates that the present theory captures the essential features of the forced receptivity process.

The most important conclusion to be drawn from these theoretical models is that significant forced receptivity occurs only when the free-stream gradients have a short scale of the same order as the instability wavelength. However, free-stream gradients that occur on the short scale of the instability wave should be viewed as a "localized" receptivity mechanism, rather than as a "distributed" mechanism. For free-stream disturbances with length scales large compared to the instability wavelength, the theories show that the receptivity due to the "distributed" mechanism becomes exponentially small. In cases of practical interest, the distance of the sources of the free-stream pressure field from the surface is usually large compared to the instability wavelength. Thus, the "distributed" mechanism involving gradients of the free-stream pressure field interacting with a smooth surface appears unlikely to be a significant source of receptivity in most practical applications. Further details on this work can be found in Ref. 17.

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4. FIGURES

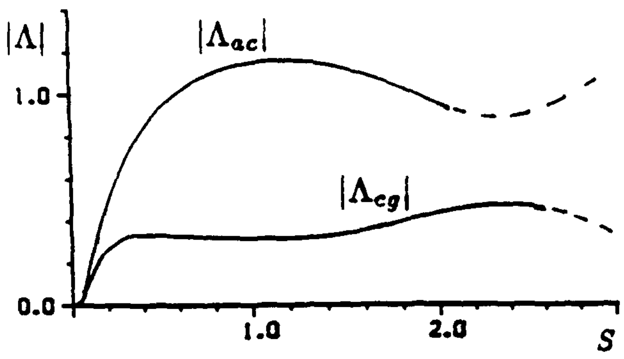

Fig. 1. The magnitudes of the efficiency functions Λ_{cg} and Λ_{ac} for interaction with a surface hump. $S = \epsilon^2 \omega L / U_\infty$

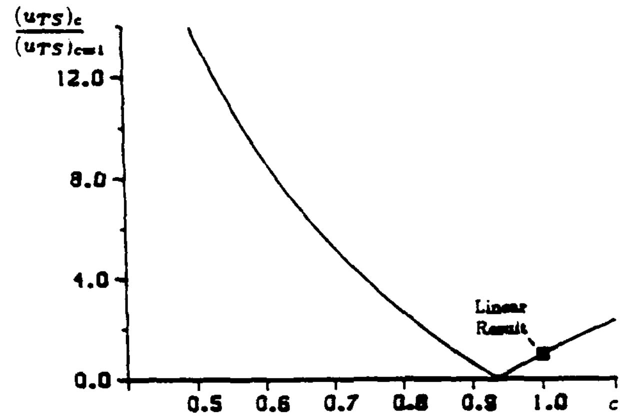

Fig. 2. Influence of convection speed U_c on the receptivity level for a Karman vortex street interacting with a wall hump. $c = U_c / U_\infty$

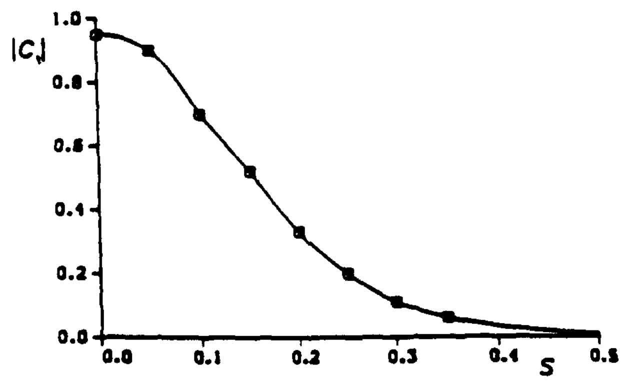

Fig. 3. Influence of body nose radius r_n on the leading-edge receptivity coefficient. $S = \omega r_n / U_\infty$

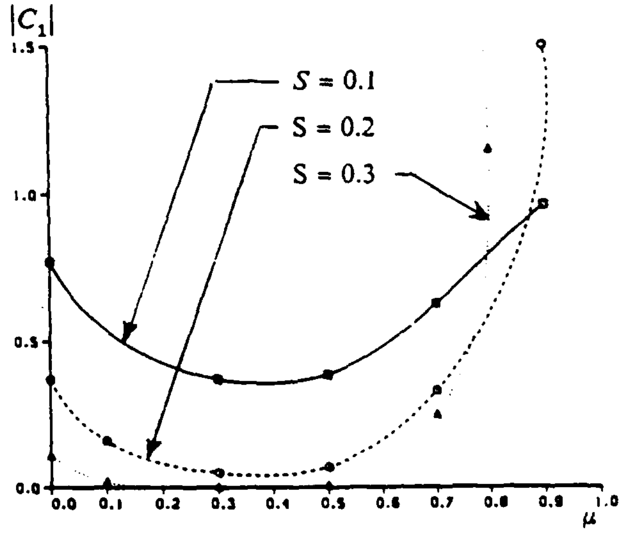

Fig. 4. Influence of aerodynamic loading on the leading-edge receptivity coefficient. $\mu = \alpha_{eff} \sqrt{b/r_n}$

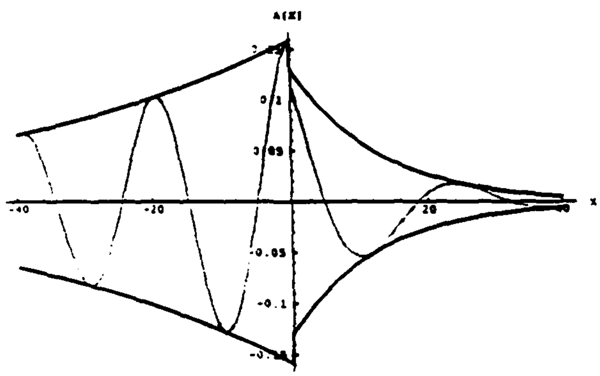

Fig. 5. Illustration of Tollmien-Schlichting wave attenuation due to mode scattering at a junction between an upstream rigid wall and a downstream admittance surface. $|\beta| = 1.3$, $S = \epsilon^2 \omega L/U_\infty = 0.6$

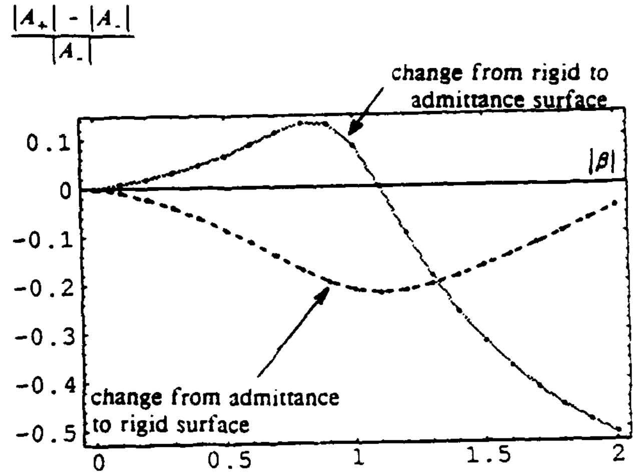

Fig. 6. Change in Tollmien-Schlichting wave amplitude due to mode scattering at a junction between a rigid wall and an admittance surface. $S = \epsilon^2 \omega L/U_\infty = 0.6$

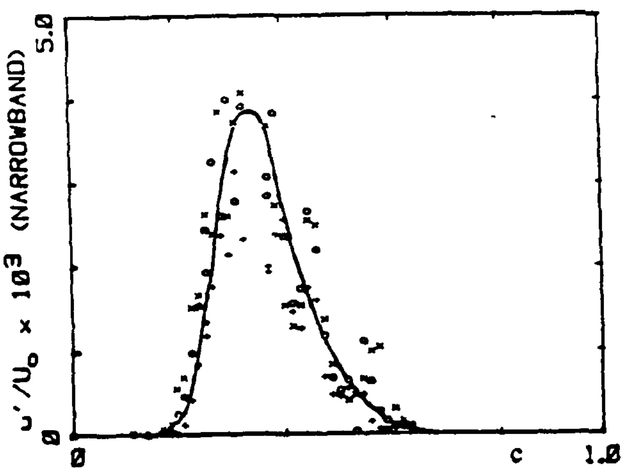

Fig. 7*a*. Kendall's experimental results for forced receptivity due to a rotating rod in the free stream. $c = U_c/U_\infty$

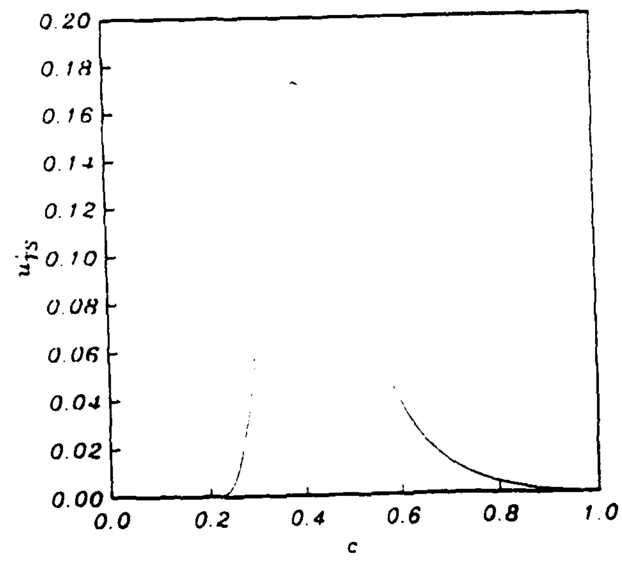

Fig. 7*b*. Theoretical prediction of forced receptivity level for Kendall's rotating rod experiment. $c = U_c/U_\infty$

5. PUBLICATIONS

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6. PARTICIPANTS

Principal Investigator:	Edward J. Kerschen, Associate Professor, Aerospace and Mechanical Engineering, University of Arizona.
Graduate Student:	Hu Zhang, Ph.D. student, Program in Applied Mathematics, University of Arizona (Ph.D. degree expected Summer 1993).

7. MEETINGS AND SEMINARS

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